PERGAMON

International Journal of Solids and Structures 36 (1999) 1369–1382

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

A two-dimensional thermoelasticity problem for a half space subjected to heat sources

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Received 30 April 1997; in revised form 23 December 1997

Abstract

The two-dimensional problem for a half space whose surface is traction free and subjected to the effects of heat sources is considered within the context of the theory of thermoelasticity with two relaxation times. Laplace and Fourier transform techniques are used. The solution in the transformed domain is obtained by using a direct approach. Numerical inversion of both transforms is carried out to obtain the temperature, stress and displacement distributions in the physical domain. Numerical results are represented graphically and discussed.

1. Introduction

During the second half of the twentieth century, non-isothermal problems of the theory of elasticity became increasingly important. This is due mainly to their many applications in widely diverse fields. First, the high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses, reducing the strength of the aircraft structure. Secondly, in the nuclear field, the extremely high temperatures and temperature gradients originating inside nuclear reactors influence their design and operations Nowinski (1978).

Danilovskaya (1950) was the first to solve an actual problem in the theory of elasticity with non uniform heat. The problem was for a half-space subjected to a thermal shock in the context of what became known as the theory of uncoupled thermoelasticity. In this theory the temperature is governed by a parabolic partial differential equation which does not contain any elastic terms. It was not much later that many attempts were made to remedy the shortcomings of this theory.

Biot (1956) formulated the theory of coupled thermoelasticity to eliminate the paradox inherent in the classical uncoupled theory that elastic changes have no effect on the temperature. The heat equations for both theories, however, are of the diffusion type predicting infinite speeds of propagation for heat waves contrary to physical observations.

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0020–7683/99/$ - see front matter
PII: S0020–7683(98)00019–5

Lord and Shulman (1967) introduced the theory of generalized thermoelasticity with one relaxation time for the special case of an isotropic body. This theory was extended by Dhaliwal and Sherief (1980) to include the anisotropic case. In this theory a modified law of heat conduction including both the heat flux and its time derivative replaces the conventional Fourier's law. The heat equation associated with this theory is hyperbolic and hence eliminates the paradox of infinite speeds of propagation inherent in both the uncoupled and the coupled theories of thermoelasticity. Uniqueness of solution for this theory was proved under different conditions by Ignaczak (1979, 1982) by Sherief and Dhaliwal (1980), by Dhaliwal and Sherief (1980) and by Sherief (1987). The state space approach to this theory was developed by Anwar and Sherief (1988) and by Sherief (1993). The fundamental solution for this theory was obtained by Sherief (1986). Sherief and Hamza (1994) have solved a two-dimensional problem for a thick layer. Sherief and Hamza (1996) have obtained the solution for two-dimensional axisymmetric problems in spherical regions and studied wave propagation in this theory.

Green and Lindsay (1972) developed the theory of thermoelasticity with two relaxation times which is based on a generalized inequality of thermodynamics. This theory does not violate Fourier's law of heat conduction when the body under consideration has a center of symmetry. In this theory both the equations of motion and of heat conduction are hyperbolic but the equation of motion is modified and differs from that of the coupled thermoelasticity theory. This theory was initiated by Müller (1971). It was further extended by Green and Laws (1972). The final form used in the present work is that of Green and Lindsay (1972). This theory was also obtained independently by Şuhubi (1975). Longitudinal wave propagation for this theory was studied by Erbay and Şuhubi (1986). Ignaczak (1978) proved a decomposition theorem. Sherief (1992) obtained the fundamental solution for this theory, formulated the state space approach for one-dimensional problems Sherief (1993) and solved a one-dimensional thermo-mechanical shock problem Sherief (1994). The boundary integral equation formulation was done by Anwar and Sherief (1994).

In this work the authors consider a two-dimensional problem for a half space whose surface is traction free and subjected to the effects of heat sources varying in time within the context of the theory of thermoelasticity with two relaxation times. Laplace and Fourier transform techniques are used. The solution in the transformed domain is obtained by using a direct approach without the customary use of potential functions. This models the industrial problem of welding of very thick plates by applying heat. It also models the effects of a thermal bomb (e.g. nuclear) on the surface of an elastic medium of a very great extent.

2. Formulation of the problem

We shall consider a homogeneous isotropic thermoelastic solid occupying the half-space $y \geqslant 0$. The y-axis is taken perpendicular to the bounding plane pointing inwards. We shall also assume that the initial state of the medium is quiescent. The surface of this medium is traction free and subjected to a heat source of intensity $r(x, t)$. The displacement vector thus, has components

$$\mathbf{u} = (u, v, 0)$$

and the cubical dilatation e is given by

$$e = \operatorname{div} \mathbf{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}. \tag{1}$$

The governing equations for generalized thermoelasticity with two relaxation times consist of:

(1) The equation of motion in vector form

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = (\lambda + \mu) \operatorname{grad} e + \mu \nabla^2 \mathbf{u} - \gamma \operatorname{grad} \left(T + v \frac{\partial T}{\partial t} \right), \tag{2}$$

which has two Cartesian components

$$\rho \frac{\partial^2 u}{\partial t^2} = (\lambda + \mu) \frac{\partial e}{\partial x} + \mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2} - \gamma \left(\frac{\partial T}{\partial x} + v \frac{\partial^2 T}{\partial x \, \partial t} \right), \tag{3}$$

$$\rho \frac{\partial^2 v}{\partial t^2} = (\lambda + \mu) \frac{\partial e}{\partial y} + \mu \frac{\partial^2 v}{\partial x^2} + \mu \frac{\partial^2 v}{\partial y^2} - \gamma \left(\frac{\partial T}{\partial y} + v \frac{\partial^2 T}{\partial y \, \partial t} \right), \tag{4}$$

where λ, μ are Lamé's constants, ρ is the density, t is the time variable, T is the absolute temperature of the medium, v is a constant with dimensions of time, called a relaxation time, and γ is a material constant given by $\gamma = (3\lambda + 2\mu)\alpha_t$, α_t being the coefficient of linear thermal expansion.

(2) The generalized equation of heat conduction

$$k \nabla^2 T = \rho c_{\mathrm{E}} \left(\frac{\partial T}{\partial t} + \tau \frac{\partial^2 T}{\partial t^2} \right) + \gamma T_0 \frac{\partial e}{\partial t}, \tag{5}$$

where k is the thermal conductivity of the medium, T_0 is a reference temperature, c_{E} is the specific heat at constant strain and τ is another relaxation time.

Applying the div operator to both sides of eqn (2), we obtain

$$\rho \frac{\partial^2 e}{\partial t^2} = (\lambda + 2\mu) \nabla^2 e - \gamma \nabla^2 \left(T + v \frac{\partial T}{\partial t} \right), \tag{6}$$

where ∇^2 is Laplace's operator.

The above equations are supplemented with the constitutive equations

$$\sigma_{xx} = (\lambda + 2\mu) e - 2\mu \frac{\partial v}{\partial y} - \gamma \left(T - T_0 + v \frac{\partial T}{\partial t} \right), \tag{7a}$$

$$\sigma_{yy} = (\lambda + 2\mu) e - 2\mu \frac{\partial u}{\partial x} - \gamma \left(T - T_0 + v \frac{\partial T}{\partial t} \right), \tag{7b}$$

$$\sigma_{zz} = \lambda e - \gamma \left(T - T_0 + v \frac{\partial T}{\partial t} \right), \tag{7c}$$

$$\sigma_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tag{7d}$$

$$\sigma_{xz} = \sigma_{yx} = 0. \tag{7e}$$

We shall use the following non-dimensional variables

$$x' = c_1\eta x; \quad y' = c_1\eta y; \quad t' = c_1^2\eta t; \quad u' = c_1\eta u; \quad v' = c_1\eta v;$$

$$\tau' = c_1^2\eta\tau; \quad \nu' = c_1^2\eta\nu; \quad \theta = \frac{T-T_0}{T_0}; \quad \sigma'_{ij} = \frac{\sigma_{ij}}{\mu}$$

where $\eta = (\rho c_E/k)$ and $c_1 = \sqrt{(\lambda+2\mu/\rho)}$ is the speed of propagation of isothermal elastic waves.

Equation (1) retains its form, while eqns (3)–(6) take the following form (dropping the primes for convenience).

$$\beta^2\frac{\partial^2 u}{\partial t^2} = (\beta^2-1)\frac{\partial e}{\partial x} + \nabla^2 u - b\left(\frac{\partial\theta}{\partial x} + \nu\frac{\partial^2\theta}{\partial x\,\partial t}\right), \tag{8}$$

$$\beta^2\frac{\partial^2 v}{\partial t^2} = (\beta^2-1)\frac{\partial e}{\partial y} + \nabla^2 v - b\left(\frac{\partial\theta}{\partial y} + \nu\frac{\partial^2\theta}{\partial y\,\partial t}\right), \tag{9}$$

$$\nabla^2\theta = \frac{\partial\theta}{\partial t} + \tau\frac{\partial^2\theta}{\partial t^2} + g\frac{\partial e}{\partial t}, \tag{10}$$

$$\beta^2\frac{\partial^2 e}{\partial t^2} = \nabla^2 e - b\nabla^2\left(\theta + \nu\frac{\partial\theta}{\partial t}\right), \tag{11}$$

where

$$b = \frac{\gamma T_0}{\mu}, \quad \beta^2 = \frac{\lambda+2\mu}{\mu} \quad \text{and} \quad g = \frac{\gamma}{k\eta}.$$

The constitutive eqns (7) in non-dimensional form can be written as

$$\sigma_{xx} = \beta^2 e - 2\frac{\partial v}{\partial y} - b\left(\theta + \nu\frac{\partial\theta}{\partial t}\right), \tag{12a}$$

$$\sigma_{yy} = \beta^2 e - 2\frac{\partial u}{\partial x} - b\left(\theta + \nu\frac{\partial\theta}{\partial t}\right), \tag{12b}$$

$$\sigma_{zz} = (\beta^2-2)e - b\left(\theta + \nu\frac{\partial\theta}{\partial t}\right) \tag{12c}$$

$$\sigma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}. \tag{12d}$$

The above equations are solved subject to the initial conditions

$$\theta = u = v = 0; \quad \dot{\theta} = \dot{u} = \dot{v} = 0 \quad \text{at } t = 0, \tag{13}$$

and the boundary conditions

$$\sigma_{yy} = 0, \quad \sigma_{xy} = 0 \quad \text{at } y = 0 \tag{14}$$

$$q_n + h\theta = r(x, t) \quad \text{at } y = 0 \tag{15}$$

where q_n denotes the normal component of the heat flux vector, h is Biot's number, and $r(x, t)$ represents the intensity of the applied heat source.

Using Fourier's law of heat conduction in non-dimensional form (which is valid for Green-theory), namely

$$q_n = -\frac{\partial \theta}{\partial n},$$

the condition (15) transforms to

$$-\frac{\partial \theta}{\partial y} + h\theta = r(x, t) \quad \text{at } y = 0. \tag{16}$$

3. Solution in the transformed domain

Taking the Laplace transform with parameter s (denoted by a bar) of both sides of eqns (8)–(12), we obtain the following set of equations

$$(\nabla^2 - s - \tau s^2)\bar{\theta} = gs\bar{e}, \tag{17}$$

$$(\nabla^2 - s^2)\bar{e} = \frac{b}{\beta^2}(1 + vs)\nabla^2\bar{\theta}, \tag{18}$$

$$\beta^2 s^2 \bar{u} = (\beta^2 - 1)\frac{\partial \bar{e}}{\partial x} + \nabla^2 \bar{u} - b(1 + vs)\frac{\partial \bar{\theta}}{\partial x}, \tag{19}$$

$$\beta^2 s^2 \bar{v} = (\beta^2 - 1)\frac{\partial \bar{e}}{\partial y} + \nabla^2 \bar{v} - b(1 + vs)\frac{\partial \bar{\theta}}{\partial y}, \tag{20}$$

$$\bar{\sigma}_{xx} = \beta^2 \bar{e} - 2\frac{\partial \bar{v}}{\partial y} - b(1 + vs)\bar{\theta}, \tag{21a}$$

$$\bar{\sigma}_{yy} = \beta^2 \bar{e} - 2\frac{\partial \bar{u}}{\partial x} - b(1 + vs)\bar{\theta}, \tag{21b}$$

$$\bar{\sigma}_{xy} = \frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x}, \tag{21c}$$

and the Laplace transform of the cubical dilatation (1) becomes

$$\bar{e} = \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y}. \tag{22}$$

We have assumed that the Laplace Transforms of the temperature, stress components, displacements, and the applied heat sources exist.

We now introduce the exponential Fourier transform (denoted by an asterisk) with respect to the space variable x, defined by

$$f^*(q, y, s) = \mathscr{T}[f(x, y, s)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-iqx} f(x, y, s)\, \mathrm{d}x,$$

with its corresponding inversion formula

$$f(x, y, s) = \mathscr{T}^{-1}[f^*(q, y, s)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{iqx} f^*(q, y, s)\, \mathrm{d}q \quad \text{where } i = \sqrt{-1}.$$

We assume that all the relevant functions (temperature, stress, ..., etc.) are sufficiently smooth on the real line such that the Fourier transforms of these functions exist.

Taking the Fourier transform of both sides of eqns (17)–(22), we obtain the following set of equations

$$(D^2 - q^2 - s - \tau s^2)\bar{\theta}^* = gs\bar{e}^*, \tag{23}$$

$$(D^2 - q^2 - s^2)\bar{e}^* = \frac{b}{\beta^2}(1 + \nu s)(D^2 - q^2)\bar{\theta}^*, \tag{24}$$

$$(D^2 - \alpha^2)\bar{u}^* = iq[b(1 + \nu s)\bar{\theta}^* - (\beta^2 - 1)\bar{e}^*], \tag{25}$$

$$(D^2 - \alpha^2)\bar{v}^* = b(1 + \nu s)D\bar{\theta}^* - (\beta^2 - 1)D\bar{e}^*, \tag{26}$$

$$\bar{\sigma}^*_{xx} = \beta^2 \bar{e}^* - 2D\bar{v}^* - b(1 + \nu s)\bar{\theta}^*, \tag{27a}$$

$$\bar{\sigma}^*_{yy} = \beta^2 \bar{e}^* - 2iq\bar{u}^* - b(1 + \nu s)\bar{\theta}^*, \tag{27b}$$

$$\bar{\sigma}^*_{xy} = D\bar{u}^* + iq\bar{v}^*, \tag{27c}$$

$$\bar{e}^* = D\bar{v}^* + iq\bar{u}^*, \tag{28}$$

where $D = (\partial/\partial y)$, and $\alpha = \sqrt{\beta^2 s^2 + q^2}$.

Eliminating $\bar{e}^*$ between eqns (23) and (24), we obtain

$$\{D^4 - [s^2(1 + \tau + \varepsilon\nu) + s(1 + \varepsilon) + 2q^2]D^2 + \tau s^4 + s^3 + s^2 q^2(1 + \tau + \varepsilon\nu) + sq^2(1 + \varepsilon) + q^4\}\bar{\theta}^* = 0, \tag{29}$$

where $\varepsilon = (bg/\beta^2)$.

The solution of eqn (29) which is bounded at infinity can be written as

$$\bar{\theta}^* = A(k_1^2 - q^2 - s^2)e^{-k_1 y} + B(k_2^2 - q^2 - s^2)e^{-k_2 y}, \tag{30}$$

where A and B are some parameters depending on q and s, and k_1 and k_2 are the roots with positive real parts of the characteristic equation

$$k^4-[s^2(1+\tau+\varepsilon v)+s(1+\varepsilon)+2q^2]k^2+\tau s^4+s^3+s^2q^2(1+\tau+\varepsilon v)+sq^2(1+\varepsilon)+q^4=0. \tag{31}$$

k_1 and k_2 are given by

$$k_{1,2}^2=q^2+\tfrac{1}{2}\{s^2(1+\tau+\varepsilon v)+s(1+\varepsilon)\pm s\sqrt{[s(1+\tau+\varepsilon v)+1+\varepsilon]^2-4s(1+\tau s)}\}. \tag{32}$$

Substituting from eqn (30) into eqn (23), we obtain

$$\bar{e}^*=\frac{b}{\varepsilon s\beta^2}\{A(k_1^2-q^2-s-\tau s^2)(k_1^2-q^2-s^2)e^{-k_1y}+B(k_2^2-q^2-s-\tau s^2)(k_2^2-q^2-s^2)e^{-k_2y}\}. \tag{33}$$

Now, since both $\bar{\theta}^*$ and $\bar{e}^*$ satisfy eqn (24), we obtain the compatibility conditions

$$(k_1^2-q^2-s-\tau s^2)(k_1^2-q^2-s^2)=\varepsilon s(1+vs)(k_1^2-q^2), \tag{34}$$

and

$$(k_2^2-q^2-s-\tau s^2)(k_2^2-q^2-s^2)=\varepsilon s(1+vs)(k_2^2-q^2). \tag{35}$$

Using eqns (34) and (35), we can write eqn (33) in the simplified form

$$\bar{e}^*=\frac{b}{\beta^2}(1+vs)[A(k_1^2-q^2)e^{-k_1y}+B(k_2^2-q^2)e^{-k_2y}]. \tag{36}$$

Substituting from eqns (30) and (36) into eqn (25), then solving the resulting equation, we obtain

$$\bar{u}^*=\frac{ibq}{\beta^2}(1+vs)[Ce^{-\alpha y}+Ae^{-k_1y}+Be^{-k_2y}], \tag{37}$$

where C is a parameter depending on q and s. Substituting from eqns (30) and (36) into eqn (28) and solving the resulting equation, we get

$$\bar{v}^*=\frac{-b}{\beta^2}(1+vs)\left[\frac{q^2}{\alpha}Ce^{-\alpha y}+k_1Ae^{-k_1y}+k_2Be^{-k_2y}\right]. \tag{38}$$

Substituting from eqns (30), (36), (37) and (38) into eqns (27), we obtain

$$\bar{\sigma}_{xx}^*=\frac{b(1+vs)}{\beta^2}[(\beta^2s^2-2k_1^2)Ae^{-k_1y}+(\beta^2s^2-2k_2^2)Be^{-k_2y}-2q^2Ce^{-\alpha y}], \tag{39a}$$

$$\bar{\sigma}_{yy}^*=\frac{b(1+vs)}{\beta^2}[(\alpha^2+q^2)Ae^{-k_1y}+(\alpha^2+q^2)Be^{-k_2y}+2q^2Ce^{-\alpha y}], \tag{39b}$$

$$\bar{\sigma}_{xy}^*=\frac{-iqb(1+vs)}{\beta^2}[2k_1Ae^{-k_1y}+2k_2Be^{-k_2y}+\frac{1}{\alpha}(\alpha^2+q^2)Ce^{-\alpha y}]. \tag{39c}$$

Taking the Laplace and Fourier Transforms of eqns (14) and (16), respectively, we obtain the boundary conditions in the transformed domain as

$$\bar{\sigma}_{yy}^*=0 \quad \text{at } y=0, \tag{40}$$

$$\bar{\sigma}^*_{xy} = 0 \quad \text{at } y = 0, \tag{41}$$

$$-D\bar{\theta}^* + h\bar{\theta}^* = \bar{r}^*(q,s) \quad \text{at } y = 0. \tag{42}$$

Equations (30), (39a) and (39b) together with eqns (40)–(42) give

$$(\alpha^2+q^2)A+(\alpha^2+q^2)B+2q^2C = 0. \tag{43}$$

$$2k_1A+2k_2B+\frac{1}{\alpha}(\alpha^2+q^2)C = 0. \tag{44}$$

$$(h+k_1)(k_1^2-q^2-s^2)A+(h+k_2)(k_2^2-q^2-s^2)B = \bar{r}^*(q,s). \tag{45}$$

Equations (43)–(45) can be solved for the three unknowns A; B and C. These solutions are

$$A = \frac{1}{\Delta}[(\alpha^2+q^2)^2-4\alpha k_2q^2]\bar{r}^*(q,s), \tag{46}$$

$$B = \frac{-1}{\Delta}[(\alpha^2+q^2)^2-4\alpha k_1q^2]\bar{r}^*(q,s), \tag{47}$$

$$C = \frac{2\alpha}{\Delta}(k_2-k_1)(\alpha^2+q^2)\bar{r}^*(q,s), \tag{48}$$

where

$$\Delta = (h-k_1)(k_1^2-q^2-s^2)[(\alpha^2+q^2)^2-4\alpha k_2q^2]-(h-k_2)(k_2^2-q^2-s^2)[(\alpha^2+q^2)^2-4\alpha k_1q^2].$$

4. Inversion of the double transforms

We shall now outline the numerical inversion method used to find the solution in the physical domain.

Let $\bar{f}^*(q,y,s)$ be the double Fourier–Laplace transform of a function $f(x,y,t)$. First, we use the inversion formula of the Fourier transform mentioned earlier to obtain a Laplace transform expression $\bar{f}(x,y,s)$ of the form

$$\bar{f}(x,y,s) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{iqx}\bar{f}^*(q,y,s)\,\mathrm{d}q = \sqrt{\frac{2}{\pi}}\int_0^{\infty}(\cos(qx)\bar{f}^*_e(q,y,s)+\sin(qx)\bar{f}^*_o(q,y,s))\,\mathrm{d}q,$$

where $\bar{f}^*_e$ and $\bar{f}^*_0$ denote the even and odd parts of $\bar{f}^*(q,y,s)$, respectively.

The inversion formula for Laplace transforms can be written as

$$f(x,y,t) = \frac{1}{2\pi i}\int_{d-i\infty}^{d+i\infty} e^{st}\bar{f}(x,y,s)\,\mathrm{d}s$$

where d is an arbitrary real number greater than all the real parts of the singularities of $\bar{f}(x,y,s)$. Taking $s = d+iy$, the above integral takes the form

$$f(x,y,t) = \frac{e^{dt}}{2\pi}\int_{-\infty}^{\infty} e^{ity}\bar{f}(x,y,d+iy)\,\mathrm{d}y.$$

Expanding the function $h(x,y,t) = \exp(-dt)f(x,y,t)$ in a Fourier series in the interval $[0,2T]$, we obtain the approximate formula Honig and Hirdes (1984)

$$f(x,y,t) = f_\infty(x,y,t) + E_\mathrm{D},$$

where

$$f_\infty(x,y,t) = \frac{1}{2}c_0 + \sum_{k=1}^{\infty} c_k \quad \text{for } 0 \leqslant t \leqslant 2T, \tag{49}$$

and

$$c_k = \frac{e^{dt}}{T} Re[e^{ik\pi t/T}\bar{f}(x,y,d+ik\pi/T)], \tag{50}$$

E_D, the discretization error, can be made arbitrarily small by choosing d large enough Honig and Hirdes (1984).

Since the infinite series in eqn (49) can only be summed up to a finite number N of terms, the approximate value of $f(x,y,t)$ becomes

$$f_N(x,y,t) = \frac{1}{2}c_0 + \sum_{k=1}^{N} c_k \quad \text{for } 0 \leqslant t \leqslant 2T. \tag{51}$$

Using the above formula to evaluate $f(x,y,t)$, we introduce a truncation error E_T that must be added to the discretization error to produce the total approximation error.

Two methods are used to reduce to total error. First, the "Korrecktur" method is used to reduce the discretization error. Next, the ε-algorithm is used to reduce the truncation error and hence to accelerate convergence.

The Korrecktur method uses the following formula to evaluate the function $f(x,y,t)$:

$$f(x,y,t) = f_\infty(x,y,t) - e^{-2dT}f_\infty(x,y,2T+t) + E'_\mathrm{D},$$

where the discretization errors $|E'_\mathrm{D}| \ll [E_\mathrm{D}|$ Honig and Hirdes (1984). Thus, the approximate value of $f(x,y,t)$ becomes

$$f_{NK}(x,y,t) = f_N(x,y,t) - e^{-2dT}f_{N'}(x,y,2T+t), \tag{52}$$

where N' is an integer less than N.

We shall now describe the ε-algorithm that is used to accelerate the convergence of the series in equation (49). Let $N = 2q+1$ where q is a natural number, and let

$$s_m = \sum_{k=1}^{m} c_k$$

be the sequence of partial sums of eqn (51), we define the ε-sequence by

$$\varepsilon_{0,m} = 0, \varepsilon_{1,m} = s_m$$

and

$$\varepsilon_{p+1,m} = \varepsilon_{p-1,m+1} + 1/(\varepsilon_{p,m+1} - \varepsilon_{p,m}), \quad p = 1, 2, 3, \ldots.$$

It can be shown that Honig and Hirdes (1984), the sequence

$$\varepsilon_{1,1}, \quad \varepsilon_{3,1}, \quad \varepsilon_{5,1}, \ldots, \varepsilon_{N,1}$$

converges to $f(x, y, t) + E_D - c_0/2$ faster than the sequence of partial sums

$$s_m, \quad m = 1, 2, 3, \ldots.$$

The actual procedure used to invert the Laplace transforms consists of using eqn (52) together with the ε-algorithm. The values of d and T are chosen according to the criteria outlined in Honig and Hirdes (1984).

5. Numerical results

The function $r(x, t)$ representing the applied heat source was taken as

$$r(x, t) = H(a - |x|)$$

where H is the Heaviside unit step function and a is a constant. This means that the applied heat source acts only on a band of width $2a$ centred around the x-axis on the surface of the half-space and is zero everywhere else.

The copper material was chosen for purposes of numerical evaluations. The constants of the problem were taken as

$$\varepsilon = 0.0168, \quad \tau = v = 0.02, \quad a = 0.1, \quad h = 1.$$

The computations were carried out for two values of time, namely $t = 0.05$ and $t = 0.1$. The numerical technique outlined above was used to obtain the temperature, displacement and stress distributions. In all figures, solid lines represent the function when $t = 0.1$, while dotted lines represent the function when $t = 0.05$. First, all the functions were evaluated inside the medium on the y-axis ($x = 0$) as functions of y. The temperature increment θ is represented by the graph in Fig. 1. The stress component σ_{xx} and σ_{yy} are shown in Figs 2 and 3, respectively. The displacement component v is shown in Fig. 4. We note that, due to symmetry, the stress σ_{xy} and the displacement component u vanish identically on the y-axis. Next, the functions were evaluated on the surface of the half-space ($y = 0$) as functions of x. The temperature increment θ on the surface is shown in Fig. 5. The stress component σ_{xx} is shown in Fig. 6. The displacement components u and v are represented in Figs 7 and 8, respectively. The stress components σ_{xy} and σ_{yy} vanish identically on the surface due to the boundary conditions.

In all these figures, it is clear that all the functions considered have a non-zero value only in a bounded region of space and vanish identically outside this region. This region expands with the passage of time. The edge of this region is the wave front which moves with a finite speed. This is not the case for the coupled and uncoupled theories of thermoelasticity where an infinite speed of

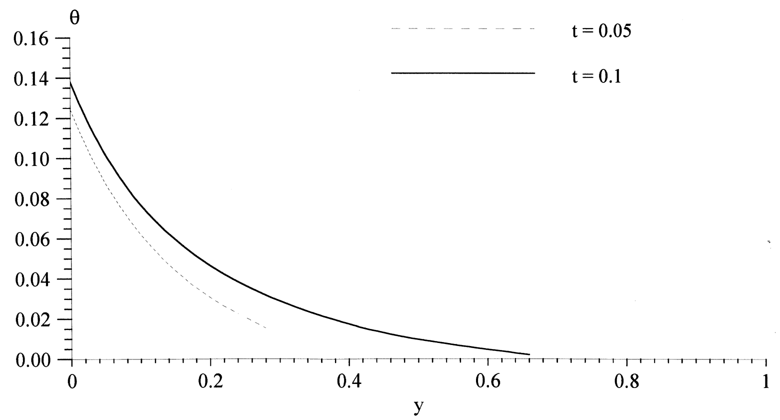

Fig. 1. Temperature distribution on the y-axis.

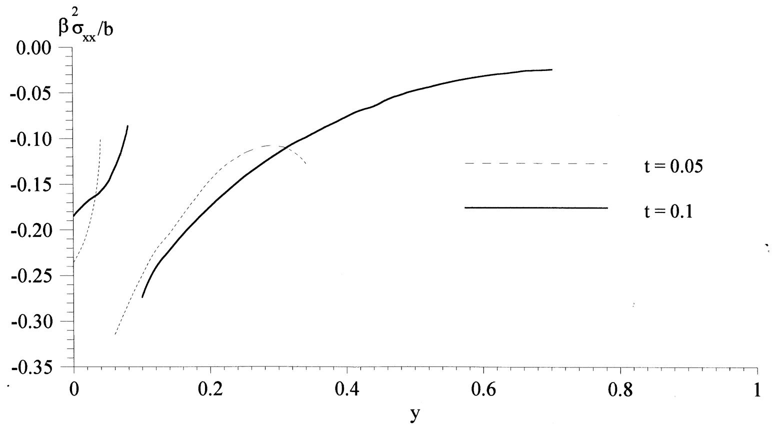

Fig. 2. Tangential stress distribution on the y-axis.

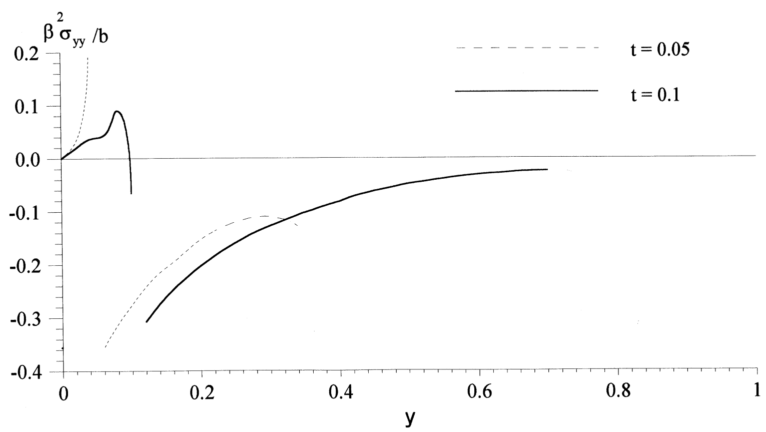

Fig. 3. Normal stress distribution on the y-axis.

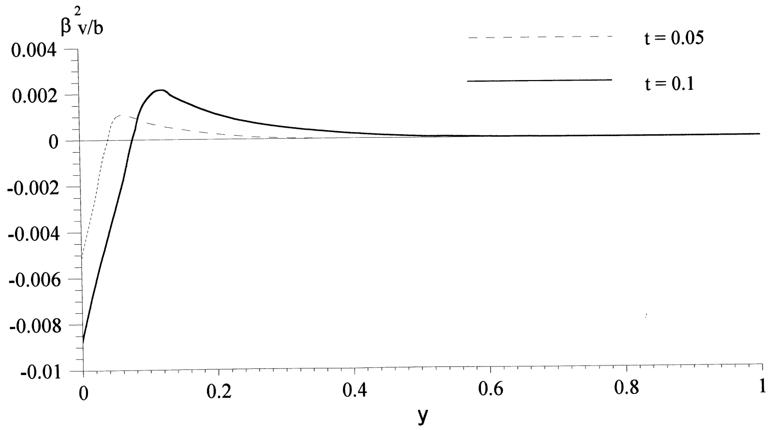

Fig. 4. Normal displacement distribution on the *y*-axis.

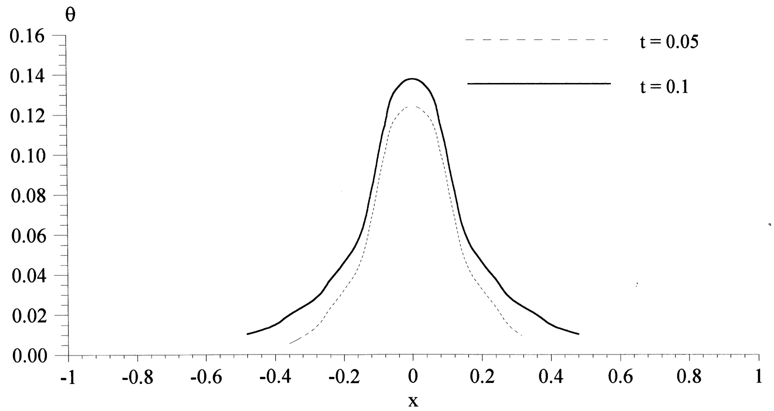

Fig. 5. Temperature distribution on the surface.

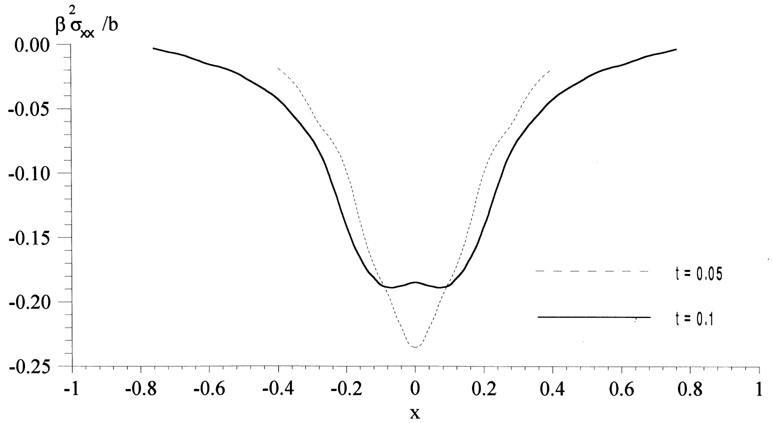

Fig. 6. Tangential stress distribution on the surface.

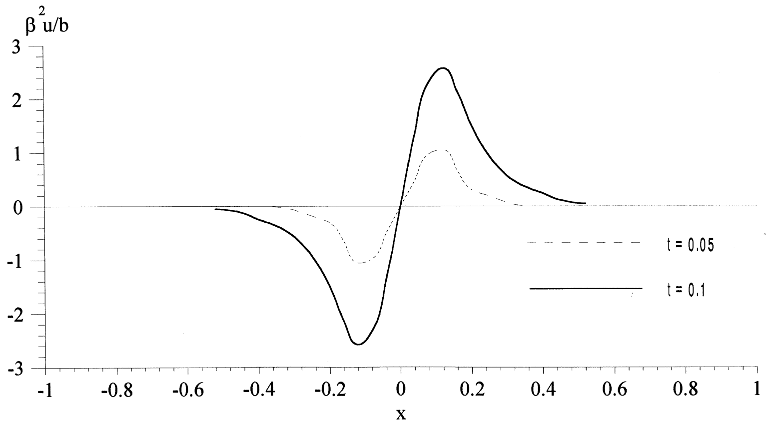

Fig. 7. Tangential displacement distribution on the surface.

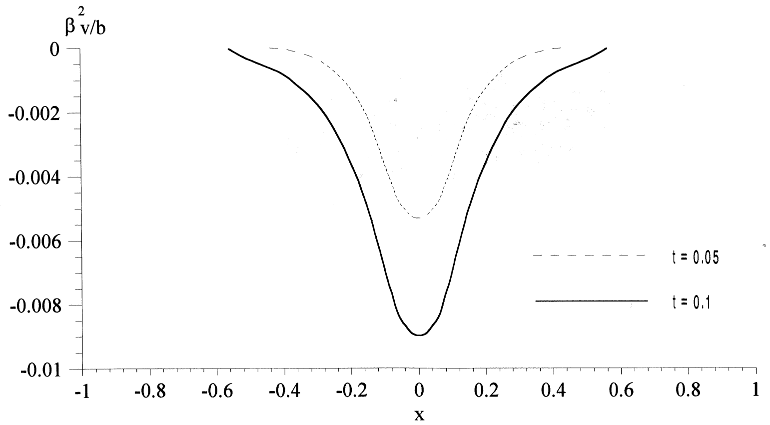

Fig. 8. Normal displacement distribution on the surface.

propagation is inherent and hence all the considered functions have a non-zero (although may be very small) value for any point in the medium.

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